WEST UNIVERSITY OF TIMIŞOARA
MATHEMATICS

ON THE ASYMPTOTIC BEHAVIOUR OF CERTAIN CLASSES OF EVOLUTION EQUATIONS

ABSTRACT OF THE HABILITATION THESIS

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ABSTRACT

The current thesis contains some of the most important results the author published pertaining to the stability and dichotomy of evolution families.

The first part of the thesis, entitled *Main scientific achievements* is structured into four chapters, the first of which details the author′s academic background, a summary of the doctoral thesis and insights into the postdoctoral research of the author.

The second chapter, entitled *On the stability of semilinear evolution equations* continues the approach initiated by Perron, [97], for semilinear nonautonomous evolution equations of the form

$$\dot{x}(t) = A(t)x(t) + f(t, x(t)) \ .$$

We assume that the linear part of the previous equation is well-posed (i.e. there exists a continuous linear evolution family $\{U(t,s)\}_{t \geq s \geq 0}$ such that for every $s \in \mathbb{R}_+$ and $x \in D(A(s))$, the function $x(t) = U(t,s)x$ is the uniquely determined solution of the corresponding linear equation satisfying $x(s) = x$). Then, we can consider the mild solution of the semilinear equation (defined on some interval $[s, s+\delta), \delta > 0$) as being the solution of the integral equation

$$x(t) = U(t,s)x + \int_s^t U(t,\tau) f(\tau, x(\tau)) d\tau \quad , \quad t \geq s \ ,$$

Furthermore, if we assume also that the nonlinear function $f(t,x)$ is jointly continuous with respect to t and x and Lipschitz continuous with respect to x, uniformly in $t \in \mathbb{R}_+$, and $f(t,0) = 0$ for all $t \in \mathbb{R}_+$), then we can generate a (non)linear evolution family $\{X(t,s)\}_{t \geq s \geq 0}$, in the sense that the map $t \mapsto X(t,s)x : [s, \infty) \to \mathbb{X}$ is the unique solution of the integral equation, for every $x \in \mathbb{X}$ and $s \in \mathbb{R}_+$.

Considering two vector-valued Schäffer function spaces $E(\mathbb{R}_+, \mathbb{X})$, $F(\mathbb{R}_+, \mathbb{X})$ satisfying a very general technical condition, and the *Green′s operator* $(\mathbb{G}f)(t) = \int_0^t X(t,s)f(s)ds$ the main theorem of the chapter shows that if some "admissibility" conditions hold then the above mild solution will have an exponential decay. The class of function spaces addressed by this approach is extremely large, since a scalar-valued Schäffer function space $E(\mathbb{R}_+, \mathbb{R})$ is any right shift invariant Banach space (actually for which the right shift is an isometry), continuously embedded in $L^1_{loc}(\mathbb{R}_+, \mathbb{R})$ that has the ideal property. There are many examples of Schäffer spaces, as for instace $L^p(\mathbb{R}_+, \mathbb{R}), M^p(\mathbb{R}_+, \mathbb{R}), p \geq 1$

(in fact any scalar-valued Orlicz function space) is in particular a Schäffer space. Also, we define the vector-valued Schäffer function space $E(\mathbb{R}_+, \mathbb{X})$ as the set of all strongly measurable $\mathbb{X}$-valued functions such that $t \mapsto \|f(t)\|$ belongs to $E(\mathbb{R}_+, \mathbb{R})$.

It is worth noting that, although the autonomous case (i.e. time invariant evolution equations), was much more analyzed than the nonautonomous case, the latter one often arises quite naturally, not only in physics and mechanics, but also in the mathematical theory of differential equations when one linearizes an autonomous equation along a nonstationary solution. For particular cases of autonomous evolution equations arising from the linearization along a compact invariant manifold, it has been shown (see e.g. [122]) that one can define a skew-product semiflow which allows to apply the methods of classical dynamical systems to the underlying time-dependent equations.

In the third chapter, *Exponential dichotomies of variational equations*, the existence of exponential dichotomies for linear skew-product semiflows (LSPS) is investigated. An exponential dichotomy is one of the most basic concepts arising in the theory of dynamical systems. This topic, for example, plays a central role in the Hadamard-Perron theory of invariant manifolds for dynamical systems, and in many aspects of the theory of stability. Even in the context of bifurcation theory, the exponential dichotomy has a role. However in this context, the exponential dichotomy is represented by its younger sibling, the exponential trichotomy. In particular, topics such as the reduction principle and the center manifold theorem, the robustness of periodic solutions and invariant manifolds, as seen in the Poincare-Melnikov scenario, are based on the theory of exponential dichotomies.

The notion of exponential dichotomy of linear differential equations was introduced by O. Perron [97]. An important contribution to these problems is the work done by J.L. Massera and J.J. Schäffer [68], J.L. Daleckij and M. G. Krein [26], W. A. Coppel [22], R.J. Sacker and G.R. Sell, [120]. The need for a new approach came from the observation that for a time dependent linear differential equation with unbounded operator $A(t)$, the solutions, generally speaking, either cannot be extended in the direction of the negative times, or can be extended, but not uniquely. All the problems above can be treated in the unified setting of a linear skew-product semiflow (LSPS). In [122], R.J. Sacker and G.R. Sell employ a notion of exponential dichotomy for skew-product semiflow with the restriction that the unstable subspace has finite dimension, and they point out a sufficient condition for the existence of exponential dichotomy for skew-product semiflow. In this chapter we use a concept of a *no past* exponential dichotomy for skew-product semiflow weaker than the concept used by Sacker-Sell. Our definition follows partially the definition (of exponential dichotomy) introduced by Chow and Leiva in [17] in the sense that we allow the unstable subspace to have infinite dimension. We go even more general and we do not assume *a priori* that the cocycle is invertible on the unstable space (actually we do not even assume that the unstable space is invariant under the cocycle). We continue the approach initiated by O. Perron (the so-called "admissibility condition" or "test function method") and we prove that the admissibility of any pair of vector-valued Schäffer function spaces (satisfying a very general technical condition) implies the existence of a (no past) exponential dichotomy. Thus, the resuslts in this chapter generalize some known results due to O. Perron [97], J. Daleckij and M. Krein [26], J. L. Massera and J.J. Schäffer [68], N. van Minh, F. Räbiger and R. Schnaubelt [82].

The aim of the fourth chapter, *On the stability of nonuniform exponential contractions*, is to obtain theorems that characterize the nonuniform exponential stability and the nonuniform exponential dichotomy for evolution families with nonuniform exponential growth (nonuniform exponential contractions as in [9], i.e. there is $M : [0, \infty) \to [1, \infty)$ such that $\|\Phi(t, t_0)\| \leq M(t_0)e^{\omega(t-t_O)}$, for each $t \geq t_0 \geq 0$).

In 1970 Richard Datko [27] succeeded in proving that the trajectories of a C_0-semigroup $\{T(t)\}_{t\geq 0}$ on a Hilbert space $\mathbb{X}$, exhibit an exponential decay if and only if they stay in $L^2(\mathbb{R}_+, \mathbb{X})$, while he attemped to extend the Lyapunov operator equation to the case of autonomous systems $\dot{x} = Ax$ with unbounded A. Two years later, A. Pazy [96] proves that the result holds if we replace $L^2(\mathbb{R}_+)$ with $L^p(\mathbb{R}_+)$, where $p \in [1, \infty)$. While Datko′s proof used the idea of a Lyapunov functional in a Hilbert space, Pazy′s proof (which is a way easier) uses the fact that $\int_0^\infty \|T(t)x\|^2 \, dt$ establishes a norm on a Banach space, and therefore we can use the pillars of functional analysis, such as the Closed Graph theorem.

In the same year, Datko extends in [28], his result to the linear nonautonomous case by proving that an evolution family $\{U(t, t_0)\}_{t\geq t_0\geq 0}$ (with uniform exponential growth) is uniform exponentially stable (i.e. there exist $N, \nu > 0$ such that $\|U(t, t_0)\| \leq Ne^{-\nu(t-t_0)}$, for all $t \geq t_0 \geq 0$) if and only if there exists $p \in [1, \infty)$ such that $\sup_{t_0\geq 0} \displaystyle\int_{t_0}^\infty \|U(t, t_0)x\|^p \, dt < \infty$ for each $x \in \mathbb{X}$. Also, the above Datko′s result for two-parameter linear evolution families was improved by Rolewicz in 1986 (see [118]). A discrete-time version of Rolewicz theorem was obtained in 1974 by Zabczyk [140] for the particular case of C_0-semigroups. Recently, it is worth to mention that the DatkoPazy theorem was generalized by van Neerven [89] in the 90s for the same particular case of C_0-semigroups.

In the final part of the thesis, the author presents a Career Development Plan containing an extensive synopsis of the targeted scientific topics where the author plans to bring his input.

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