WEST UNIVERSITY OF TIMIȘOARA
DOCTORAL SCHOOL OF EXACT SCIENCES AND NATURAL SCIENCES

PhD THESIS
Qualitative and quantitative analysis of nonlinear dynamic games with time delays

Summary

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Contents

Keywords

- Game theory
- Mathematical model
- Time delays
- Equilibrium point
- Local stability
- Discrete time delay
- Distributed time delay
- Continuous time delay
- Flip bifurcations
- Hopf bifurcations
- Nonlinear systems
- Homogeneity expectation
- Heterogeneity expectation
- Chaotic regimes

Summary

Motivation

Cournot and Bertrand duopoly games were introduced in [5] (1838) by Antoine Augustin Cournot. In 1934, von Stackelberg [25] described the Stackelberg leadership model of firm competition. The work of von Neumann [22] (1944) represents the foundation book on game theory, discussing interactions where the outcomes depend on the actions of multiple agents. In 1961, Fels [11] introduced the game theory in the oligopoly market, assuming that only a few firms dominate the market.

In a simpler setting, the Cournot duopoly model has been extended by Kopel in [17] (1996) to consider a nonlinear inverse demand function, leading to the detection of nonlinear phenomena such as bifurcation of limit values of production or deterministic chaos. Due to these theoretical developments, the study of duopoly games, such as Cournot and Bertrand models, proved to be helpful in understanding the strategic interactions between firms in competitive markets [10] (1999). In a continuous-time Cournot duopoly model, the dynamic behavior is influenced by the heterogeneity in firms' output decisions, which can be modeled using differential equations with discrete time delays, first considered by Bischi et al. in [3] (2000). In the discrete time setting, the chaotic features are identified by Agiza and Elsadany in [1] (2004).

Historically, Cournot-type models have provided essential insights into firm behavior, equilibrium formation, and market dynamics, but have often neglected the realistic factor of time delays. Recognizing that in practical scenarios, firms typically face delays due to production lags, information processing, or decision-making processes, it has been shown that incorporating time delays into duopoly models can lead to complex dynamic behaviors such as Hopf bifurcations [18] (2000). The study of discrete-time duopoly games with distributed time delays involves the analysis of strategic interactions between two firms where decisions are influenced by past information [4] (2000).

In a more complex framework, the study of oligopoly games with time delays, discrete and distributed, involves understanding how delays impact the stability and dynamics of the game. In Cournot oligopoly models, firms make decisions based on delayed information about competitors' outputs [23] (2001). The analysis of oligopoly games with distributed time delays in the context of game theory involves the study of stability, bifurcation, and chaos control, as well as the practical implications for decision-making in competitive markets. The impact of distributed time delays on the stability of the evolutionarily stable strategy in continuous-time dynamics has been studied in [12] (2005). A discrete oligopoly game involves a market structure where a few firms compete with each other and can be modeled in various ways, often using game theory to analyze the interactions between competitors. The discrete oligopoly game with time delays involves strategic interactions among firms where decisions are influenced by past actions, as seen in the works of Matsumoto and Szidarovszky [19, 20, 21]. This concept is explored in various models and scenarios, highlighting the complexity and dynamic nature of such systems [26].

The motivation for this thesis arises from the established significance of mathematical models in analyzing competitive interactions within duopoly and oligopoly markets, particularly under the Cournot framework. By explicitly incorporating time delays into various Cournot models, the research presented in this thesis explores and characterizes how such delays affect market equilibria, their stability, and complex qualitative dynamics. A rigorous mathematical analysis of stability conditions, bifurcations, and emerging behaviors is conducted, providing a deeper economic interpretation of how delayed responses influence competitive strategies and market outcomes.

Objectives

General objectives

The general objective of this thesis is to investigate discrete- and continuous-time mathematical models for Cournot duopoly and oligopoly markets, explicitly incorporating the impact of time delays. Specifically, this thesis aims to examine how time-dependent dynamics, such as production lags, delayed responses to market fluctuations, and the utilization of outdated information, affect strategic firm interactions, equilibrium stability, and overall market outcomes.

Through qualitative analysis, the thesis investigates the conditions under which equilibria remain stable or become unstable as a result of varying delay parameters. Furthermore, this work systematically explores the qualitative changes in market behavior arising from delay-induced bifurcations, extending the theoretical understanding of competitive dynamics in markets characterized by delayed decision-making processes. By combining analytical results with extensive numerical simulations, this thesis provides clear insight into the complex interplay between delays and market competition.

The results of this thesis significantly improve existing theoretical frameworks and offer valuable analytical tools to inform strategic decision making and policy formulation in oligopolistic and duopolistic market environments.

Specific Objectives

- Develop and investigate dynamic Cournot-type duopoly and oligopoly models that incorporate multiple time delays in both discrete and continuous-time frameworks, describing realistic lags in production and decision processes.
- Undertake rigorous mathematical analysis of equilibrium points, examining local stability under varying delay parameters.
- Explore the emergence of bifurcations caused by delays, determining the critical thresholds at which qualitative changes in market dynamics, such as periodicity

or chaos, occur.

- Use numerical simulations to illustrate and validate theoretical findings, highlighting how delayed feedback mechanisms can alter expected outcomes and offering concrete examples of stability loss or chaos.
- Compare standard Cournot competition with a Stackelberg–Cournot framework in the presence of delays, analyzing how sequential decision-making (leader–follower) modifies equilibrium and stability characteristics.
- Contribute to the broader theoretical development of dynamical systems and stability theory within mathematical economics by identifying the key parameters, structural conditions, and delay mechanisms that significantly influence long-term market outcomes.

Structure of the thesis and original results

This thesis analyzes mathematical models of Cournot-type duopoly and oligopoly games with distributed time delays, in both discrete and continuous time settings. By allowing each firm's current output decision to depend on the history of its own production and that of its rival, such models incorporate memory effects that influence market stability and strategic interaction between firms.

The thesis titled *"Qualitative and Quantitative Analysis of Dynamic Non-Linear Games with Time Delays"* is organized into six chapters. Chapter 1 states the motivation, the objectives, and the original contributions. Chapter 2 reviews key results on discrete and continuous-time duopoly / oligopoly games (see [1, 3, 19, 20, 21]). Chapters 3–5 are written as standalone manuscripts: Chapter 3 analyses a continuous-time Cournot duopoly with distributed delays; Chapter 4 studies three discrete-time Cournot oligopoly models with one, two and three delays; and Chapter 5 develops a Stackelberg–Cournot oligopoly with delays. Finally, conclusions are drawn in Chapter 6.

In what follows, we present a concise overview of the contents of each chapter and principal original results, highlighting both the mathematical techniques used and their economic significance.

Chapter 1 introduces the motivation, the general and specific objectives and the structure of this thesis, emphasizing the original contributions of this work. The main question of this thesis is how distributed delays alter competition in Cournot-type duopoly and oligopoly markets, a topic that combines economic relevance (predicting market stability) with mathematical interest (delay differential and difference equations).

Chapter 2 introduces the mathematical and economic background of the thesis. It begins with a short survey of non-cooperative, cooperative and Stackelberg game theory, then recalls the classical Cournot duopoly and oligopoly frameworks. Dynamic formulations in discrete and continuous time are introduced with discrete and distributed delays, and several introductory stability and bifurcation results are outlined. The discussion extends to n-firm oligopolies with product differentiation, where the best-response and partial-adjustment dynamics are analyzed. The closing sections contrast homogeneous and heterogeneous expectation regimes, preparing the ground for the models analyzed in Chapters 3–5.

Chapter 3: "A general continuous time Cournot duopoly model with distributed time delays", is based on the original paper

- [8] - *"Stability and bifurcations in a general Cournot duopoly model with distributed time delays"*, ***Chaos, Solitons*** & ***Fractals***, Vol. 162: 112424, 2022.

At the time of writing of the present thesis, this paper has already inspired further work in dynamic games with delays and related control problems, being cited by [13, 14, 15, 16, 24, 27].

This chapter presents a nonlinear continuous–time Cournot duopoly with *four distributed time delays*. Each firm reacts to the entire history of its own output and that of its rival, so two delay kernels are added to every adjustment rule. The system admits four equilibria; the three with at least one zero component are proved to be unstable for any delay kernels, whereas the interior equilibrium can switch stability via a Hopf bifurcation. Four delay configurations are analyzed in detail: (i) rival output delays only, (ii) identical kernels for both firms, (iii) delays for one firm only, and (iv) own output delays only. The analytical stability and bifurcation conditions are illustrated by numerical simulations.

The chapter is organized in the following sections. After the introductory Section 3.1, Section 3.2 formulates the continuous-time Cournot duopoly that incorporates four distributed time delays. In Section 3.3, we derive the four equilibria and perform the preliminary local stability analysis. In Section 3.4, we prove that the null equilibrium E_1 and the two boundary equilibria E_2, E_3 are unstable for all delay kernels. Section 3.5 focuses on the interior equilibrium E_4, establishing explicit conditions for asymptotic stability and the occurrence of Hopf bifurcations. Throughout, numerical simulations validate the theory under the four delay configurations described above. Finally, conclusions are drawn in Section 3.6.

The original results of this chapter are listed below. The instability of the null equilibrium E_1 and the boundary equilibria E_2, E_3 is proved in Propositions 3.1 and 3.2, regardless of the delay kernels considered in the Cournot duopoly model. In the case when only the competitors' delays are considered for both players, Proposition 3.3 proves that the positive equilibrium E_4 is asymptotically stable for any such delay kernels. If the same delay kernel is considered for both players, Proposition 3.5 represents a complete characterization of the stability and bifurcation properties in a neighborhood of the positive equilibrium E_4 and establishes the critical value of the average time delay for the occurrence of a Hopf bifurcation. If no delays are considered for one player, Proposition 3.6 provides the implicit equations of the corresponding Hopf bifurcation curves in the plane of the average time delays associated with the other player. If only own delays are considered for both players, Proposition 3.7 provides a characterization of the stability properties and Hopf bifurcation occurring in a neighborhood of the equilibrium E_4. All these results are supported by proofs, theoretical examples and numerical simulations. The author of this thesis has developed the Wolfram Mathematica code used in this chapter, which is released as open-source on GitHub [28].

Chapter 4: "Discrete-time dynamic Cournot models" is based on following three original papers, namely:

- [6] - *"Stability properties of a Cournot-type oligopoly model with time delay"*, ***Annals of West University of Timisoara - Mathematics and Computer Science***, Vol. 59, no. 1, p. $121-129$, 2023;
- [7] - *"A dynamic Cournot mixed oligopoly model with time delay for competitors"*, ***Carpathian Journal of Mathematics***, Vol. 38, no. 3, p. $681-690$, 2022;
- [9] - *"Dynamics of a discrete-time mixed oligopoly Cournot-type model with three time delays"*, ***Mathematics and Computers in Simulation***, Vol. 226, p. $524-539$, 2024.

At the time of writing, the paper [9] was cited by [2].

This chapter investigates three discrete-time Cournot–type oligopoly models, each introducing an additional time delay. Section 4.1 studies an oligopoly of n private firms whose current output decisions depend on the delayed total production of their rivals; the system admits a single interior equilibrium that is asymptotically stable under a verifiable parameter restriction. Section 4.2 turns to a *mixed* oligopoly with one public firm and n private companies, including *two* delays: the public firm responds to the past output of the private firms, while each private firm reacts to the delayed production of the public firm. Two equilibria may coexist: the boundary equilibrium is always unstable, while the interior equilibrium is locally stable only within a specific parameter region. Section 4.3 extends the same structure to *three* time delays and characterizes the stability region of the interior equilibrium, showing how time delays influence the stability region. Numerical simulations illustrate the theoretical results obtained in this chapter. Conclusions are formulated in Section 4.4.

The original findings from this chapter are presented in what follows. In the first section, the necessary and sufficient condition for the stability of the positive equilibrium in the absence of delay within the investigated oligopoly model is established in Remark 4.1. In the presence of time delay, Proposition 4.2 provides the necessary and sufficient condition for the asymptotic stability of the positive equilibrium. For a given time delay and degree of product differentiation, Remark 4.3 gives the exact formula for the maximum number of private firms considered in the model, for which the equilibrium remains asymptotically stable. In the second section, the outcome of the local stability analysis is that the boundary equilibrium E_0 is a saddle point, as proven in Theorem 4.4. In the absence of time delays, Theorem 4.5 provides necessary and sufficient conditions for the asymptotic stability of the positive equilibrium. Moreover, Theorem 4.7 gives sufficient conditions for the asymptotic stability of the positive equilibrium E_+, regardless of time delays. The occurrence of a flip bifurcation in a neighborhood of the positive equilibrium is established in Remarks 4.9 and 4.10. In the third section, in the presence of arbitrary time delays, the boundary equilibrium E_0 is shown to be a saddle point in Theorem 4.11. The delay-free stability region of the positive equilibrium E_+ is characterized in Theorem 4.12. Furthermore, Theorems 4.14 and 4.15 prove that these delay-free stability conditions guarantee the stability of the positive equilibrium if certain configurations of time delays are considered. Proposition 4.17, Remark 4.18 and Proposition 4.19 provide a characterization of the flip and Neimark-Sacker bifurcation curves, respectively, in the considered parameter space, near the positive equilibrium E_+. All results are rigorously validated through mathematical proofs, illustrative theoretical examples, and supporting numerical simulations. The Wolfram Mathematica code employed in this chapter was developed by the author of this thesis and is available as open-source on GitHub [29, 30, 31].

Chapter 5: "A discrete-time Stackelberg-Cournot oligopoly with time delays" is based on the original paper titled *"A dynamic oligopoly Stackelberg-Cournot model with time delays"*, which at the time of writing is under review.

In this chapter, we consider a discrete-time Stackelberg–Cournot oligopoly with n leader firms and m follower firms. Section 5.1 treats the delay-free system: the demand and cost functions are fixed and after the equations are non-dimensionalised, two equilibrium points are found: a boundary equilibrium and a positive (interior) equilibrium, whose local stability is shown to depend on the cost structures of leaders and followers. Section 5.2 introduces time delays into the quantity-adjustment rules and derives explicit delay-dependent stability conditions, demonstrating that the interior equilibrium can lose stability through a bifurcation, while the boundary equilibrium remains a saddle. Numerical simulations throughout the chapter confirm the analytical results and illustrate how delay and cost asymmetry influence the leaders' first-mover advantage.

The original results from this chapter are summarized bellow. In the first section, the mathematical model without delay is described. Also, this includes: demand and cost assumptions, profit maximization and best responses, discrete-time dynamical setup, nondimensionalization and equilibria, stability of the boundary and positive equilibrium are included, the second section contains the mathematical model with time delays: delayed model equations, and the stability of the equilibria. Additionally, numerical simulations validate the theoretical findings presented in this chapter.

The original results of this chapter are summarized in the following. First, two separate scenarios are distinguished: (A.1) - which corresponds to an industry structure in which there are only a few followers, and the cost ratio is moderate, and (A.2) - in which leader firms have reduced production costs with respect to the followers. In Remarks 5.1 and 5.2 we argue that the presence of a positive equilibrium E_+ is guaranteed in these economically meaningful scenarios, where both leaders and followers maintain positive production levels. The positive equilibrium coexists with a boundary equilibrium E_0, which is less relevant to a normal economic activity. In the absence of time delays, Proposition 5.3 and Remark 5.4 characterize the stability properties of the boundary equilibrium E_0, identifying it as a saddle point under assumption (A.1), while in scenario (A.2), it transitions from asymptotic stability to instability via a flip bifurcation occurring at a critical value of the characteristic parameter considered. On the other hand, in the absence of delays, Proposition 5.5 characterizes the stability properties of the positive equilibrium E_+: under assumption (A.1), it loses asymptotic stability through a flip bifurcation as the characteristic parameter reaches a critical threshold, while in scenario (A.2), it is shown to be a saddle point. When time delays are included in the mathematical model, the stability properties of the boundary equilibrium E_0 remain the same as in the non-delayed case, as shown in subsection 5.3.3. Moreover, Proposition 5.7, Remark 5.8 and Proposition 5.9 clarify the scenarios under which the stability properties of the positive equilibrium E_+ remain the same as in the non-delayed case. However, Lemma 5.11 and Proposition 5.12 show that only in scenario (A.1), time delays may modify the stability characteristics of E_+, and the boundary of the corresponding stability region may be replaced by a Neimark-Sacker bifurcation curve. Therefore, more complex quasi-periodic dynamics and route to chaotic behavior are observed. All results are supported by mathematical proofs, examples, and numerical simulations. The Python code used in this chapter, developed by the thesis author, has been released as open-source on GitHub [32].

This thesis concludes with a section of conclusions drawn in **Chapter 6**, followed by the list of original publications, the list of oral presentations at conferences, and the bibliographic references.

The main contribution of this thesis is the investigation of three delayed economic game models by combining a unified analytical approach with numerical simulations, with the aim of providing a methodological framework that other researchers can use to study how memory and implementation delays influence strategic competition.

Author’s results

Published papers

Articles published in ISI journals

1. Loredana Camelia Culda, Eva Kaslik, Mihaela Neamţu, ”Stability and bifurcations in a general Cournot duopoly model with distributed time delays”, Chaos, Solitons & Fractals 162 (2022): 112424.
2. Loredana Camelia Culda, Eva Kaslik, Mihaela Neamţu, ”A dynamic Cournot mixed oligopoly model with time delay for competitors”, Carpathian Journal of Mathematics 38, 3 (2022): 681–690.
3. Loredana Camelia Culda, Eva Kaslik, Mihaela Neamţu, Nicoleta Sîrghi, ”Dynamics of a discrete-time mixed oligopoly Cournot-type model with three time delays”, Mathematics and Computers in Simulation, 226 (2024), 524–539.

Articles published in BDI journals

- Loredana Camelia Culda, ”Stability properties of a Cournot-type oligopoly model with time delay.”Annals of West University of Timisoara - Mathematics and Computer Science, 59 (2023), 121–129.

Articles sent for publication

- Loredana Camelia Culda, Eva Kaslik, Gabriela Mircea, Mihaela Neamţu, ”A dynamic oligopoly Stackelberg-Cournot model with time delays” (in review).

Presentations at conferences and workshops

Oral presentations at conferences

- ”Cournot game with delays”, ”Current Economic Trends in Emerging and Developing Countries”, Timișoara, România, 2021;
- ”Bifurcation analysis for a discrete-time delayed dynamic Cournot mixed oligopoly model”, International Conference ”on Mathematical Analysis and Applications in Science and Engineering”, Porto, 27-29 June 2022;
- ”A dynamic Cournot oligopoly model with two time delays”, ”Mathematics - the engine of contemporary science: vision, methods, innovation”, Smart Diaspora,

Timişoara România, 10-14 April 2023.

- ”A model for competitors with time delay in a dynamic Cournot oligopoly”, International Conference ”Current Economic Trends in Emerging and Developing Countries”, Timișoara, România, 8-9 June 2023.
- ”Mixed Cournot oligopoly’, ”ICNPAA: Mathematical Problems in Engineering, Aerospace and Sciences”, Cehia, Praga, 27-30 June 2023.
- ”A mixed oligopoly model with time delays”,”International Association for Mathematics and Computers in Simulation”, Roma, 11-15 September 2023.
- ”An oligopoly model with three time delays”, ”29th International Conference on Difference Equations and Applications”, Paris, 24-28 June 2024.
- ”An oligopoly Stackelberg-Cournot model”, ”16th International Conference on Mathematics and its Applications”, Timişoara, 29-31 May 2025.

Workshop presentations

- ”Bifurcation analysis for dynamic Cournot mixed oligopoly model”, Austrian Center for Medical Innovation and Technology, Dynamics - H2020 - MSCA - RISE - 2017 - 777911, Wiener Neustadt, 16 August 2022.

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[32] https://github.com/culdana/Stackelberg-Cournot-oligopoly-game.